Professor Osborne Reynolds (Copyright, The University of Manchester)

Osborne Reynolds and the Publication of His Papers on Turbulent Flow

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Abstract

Following The Royal Society's decision to allow the release of hitherto confidential documents concerning referees' reports and other material relating to the publication of historical manuscripts, the present paper examines the exchanges preceding the publication of the two papers on turbulent flow by Osborne Reynolds that have so greatly influenced the development of Engineering Fluid Mechanics over the past century. The documents cited reveal that, although the earlier experimental paper was warmly welcomed, the referees were critical of the subsequent analytical contribution. It appears that the publication of the latter paper was due mainly to the considerable standing Reynolds had by then acquired, in part from the impact of his experimental paper published 12 years earlier. The paper also provides a summary of Reynolds' career and research prior to his embarking on the research published in the two seminal papers.

INTRODUCTION

For readers of the *Annual Review of Fluid Mechanics*, the name and contributions to Fluid Mechanics of Osborne Reynolds need no introduction. Indeed, it is no exaggeration to assert that his two principal papers on turbulent flow, "An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels," Reynolds (1883), and "On the dynamical theory of incompressible viscous fluids and the determination of the criterion," Reynolds (1895), which were both published in the *Philosophical Transactions of the Royal Society*, essentially provided a marker for the direction of research in Engineering Fluid Mechanics for the next century. But how did these papers arise? Were they subject to anything like the same peer-review procedures that academics are familiar with today?

These and attendant questions can at least now be partly answered. Some three years ago, The Royal Society decided to make available from its archives many hitherto confidential documents including referees' reports on papers published sufficiently long ago that their transfer into the public domain could have not the slightest impact on the reputation of any living person. Thus it was that, following an enquiry by the present writers, copies of the available exchanges between editor, referees, and the author of those two papers by Osborne Reynolds have been obtained. They form the main focus of this paper. First, however, we present an overview of Reynolds' family background, education, and training, his appointment to the Chair of Civil and Mechanical Engineering at Owens College, Manchester, and his career there. Thereafter, we give a brief account of some of the parallel streams of research that he pursued in the period of just over a decade, leading up to the first of his fundamental studies of turbulent flow.

BIOGRAPHICAL OVERVIEW

Osborne Reynolds was born on August 23, 1842. He came from a well established family that owned much of the land and property in the small farming community of Debach, near Woodbridge, in Suffolk. His great-grandfather and grandfather were rectors of the parish of Debach-with-Boulge, while his father, the Reverend Osborne Reynolds, was a Fellow of Queens' College, Cambridge and, at the time of his son's birth, Principal of the Belfast Collegiate School, then later Headmaster of Dedham Grammar School, Essex, and also for a short period Rector of Debach-with-Boulge. The task of providing Reynolds' early education was undertaken mainly by his father, who as well as being an extremely able mathematician had a keen interest in mechanical matters and took out several patents concerned with improvements to agricultural equipment. The titles of these can be found in the paper "The Life and Work of Osborne Reynolds" (Allen 1969).

The young Osborne Reynolds showed an early aptitude and liking for the study of mechanics and, at the age of 19, entered the apprentice school and workshop of Edward Hayes of Stony Stratford in Buckinghamshire, a well-known mechanical engineer who, as well as running a firm that manufactured agricultural machinery, steam engines, and small coastal steamers, provided technical training and practical experience for the sons of privileged families. Osborne Reynolds remained with Hayes as an apprentice for nearly two years. During this period, to use his own words, "my attention [was] drawn to various mechanical phenomena, for the explanation of which I discovered that a knowledge of mathematics was essential." He therefore decided to go to the University of Cambridge to take a degree in mathematics. His studies at Cambridge were highly successful, although Reynolds was not very happy with the manner in which mechanics was taught there, which he considered to be aimed mainly at applications in physics rather than what he considered to be more useful and urgent ones in engineering. (It is noted that the Lucasian Professor in the Department of Mathematics when Reynolds was an undergraduate was Sir George Stokes, who figures prominently in this paper at a later stage.) Osborne Reynolds graduated in 1867 and was immediately elected to a Fellowship at Queens' College. He then joined a reputable firm of civil engineering consultants, Lawson and Mansergh of London, to obtain some professional experience. They specialized in municipal engineering projects, particularly concerned with water supply, drainage, and sewage systems. Working with James Mansergh, Osborne Reynolds developed an interest in public health matters, which later led him to write a short monograph entitled "Sewer gas and how to keep it out of houses – a handbook of house drainage" Reynolds (1872), which he published a few years later after he had had an opportunity to validate the ideas contained therein by applying them himself in his own house and demonstrating their effectiveness.

Early in 1868, Osborne Reynolds applied for the newly instituted Chair of Civil and Mechanical Engineering at Owens College, Manchester, which 12 years later became The Victoria University of Manchester. In his application for the post, Osborne Reynolds stated, "From my earliest recollection I have had an irresistible liking for mechanics and the physical laws on which mechanics as a science are based. In my boyhood I had the advantage of the constant guidance of my father, also a lover of mechanics and a man of no mean attainment in mathematics and their applications to physics." The arrangements for the appointment of a professor did not go entirely smoothly. After the post was initially advertised, Owens College came under criticism partly as a result of a satirical article in the professional journal *Engineering* for not offering a sufficiently attractive salary. Even though the initial advertisement had led to a not insignificant number of applications, including some from suitably qualified candidates, in the face of this very public criticism the College decided to readvertise the post offering a higher salary that was guaranteed to be at least £500 per annum. This certainly strengthened the response. In fact, at one stage it seemed that the highly respected and well established W. J. M. Rankine, who at that time held the prestigious Chair of Engineering at the University of Glasgow, might be showing some interest in applying for the post. However, when this did not happen the appointing committee drew up a shortlist of two candidates, one of whom was Osborne Reynolds. He was duly interviewed and appointed on March 26, 1868, being only 25 years old at the time. Osborne Reynolds took up his appointment on September 29, 1868 and a few days later (October 5th) gave the customary Introductory Address to inaugurate the new academic session for 1868–1869 at Owens College. Delivered at the Royal Institution in Manchester, this lecture on "The Progress of Engineering considered with respect to the social condition of his country" was his first published work (Reynolds 1868). It gives a penetrating view of the problems of the times as he saw them and of his attitude toward engineering in relation to the work, wealth, and happiness of mankind. Made at the outset of his career, the lecture shows the youthful Reynolds clear in his mind as to what needed to be done, a figure charged with a sense of mission and full of ideas. Osborne Reynolds remained as Professor of Civil and Mechanical Engineering at the University of Manchester until his retirement in 1905 and died at Watchet in Somerset on February 21, 1912 at the age of 69.

Reynolds' considerable mathematical ability was complemented by an uncanny insight into the physical fundamentals of a problem. Shortly after coming to Manchester, he began a series of original researches that led, during the next 35 years, to the publication of many papers of outstanding interest. These covered (even by the standards of the time) an exceptionally wide range of physical problems and engineering applications. His collected work was published by Cambridge University Press in three volumes with the title *Papers on Mechanical and Physical Subjects* (Reynolds 1900, 1901, 1903). These contain most of his papers, more than 70 in all.

Osborne Reynolds received an array of academic honors during his lifetime. He was made an Honorary Fellow of Queens' College Cambridge in 1882. Before that, in 1877, he had been elected a Fellow of The Royal Society and in 1888 received the Society's Royal Medal. In 1883, he became a Member of the Institution of Civil Engineers and was awarded the Telford Premium in 1885. The University of Glasgow conferred the Honorary Degree of L.L.D. on him in 1884. Finally, he was elected President of the Manchester Literary and Philosophical Society in 1888 and received the Dalton Medal in 1903. Yet, despite this extensive institutional recognition of his outstanding academic and engineering accomplishments, he did not receive a knighthood for his contributions as the two other eminent fluid mechanicists figuring in this account were to do.

SOME EARLY STREAMS OF HIS RESEARCH

At the time Osborne Reynolds took up his post at Manchester, Owens College was housed in a relatively small building on Quay Street that had earlier been the home of a local member of Parliament. Little was available to Reynolds in the way of laboratory facilities for either teaching or research. Thus, initially, he was restricted to addressing problems involving very simple experiments that could either be done at home or outdoors. We see this clearly reflected in the emphasis of his early work. It was not until much later, well after 1873 when Owens College moved to new buildings on the present site of the University of Manchester, that he was able to undertake more elaborate experiments, and it was even later before he had laboratory facilities that enabled him to perform tests on large-scale engineering plant.

In November 1869, Osborne Reynolds became a member of the Manchester Literary and Philosophical Society. At that time the President of the Society was the distinguished Manchester scientist James Prescott Joule. It was under the latter's encouraging eye that Reynolds read his first paper to the Literary and Philosophical Society in March 1870. It was on "The stability of a ball above a jet of water" (Paper 1 in Reynolds 1900), an interesting if rather academic problem, the explanation of which involved ideas on aspects of fluid dynamics not usually thought of as dating back to that time. This first paper was followed by many others, covering an extremely wide range of physical problems and engineering applications. A detailed review of Reynolds' work is given by Jackson (1995). Below are two examples of the several diverse streams of research that he pursued during the initial decade or so of his working life at Manchester, leading up to his contributions on turbulent flow that are of central interest here.

Natural Phenomena (1870–1880)

This early stream of research, which was later described by his most famous student, J.J. Thomson, as "Out of doors Physics," was particularly convenient as an area of research interest in view of the lack of laboratory facilities in Owens College at that time. His papers on such phenomena, which are all found in Volume I of his collected work (Reynolds 1900), deal with matters concerned with comets, the solar corona, terrestrial magnetism, the electrical properties of clouds, the bursting of trees struck by lightning, the destruction of sound by fog, the refraction of sound by the atmosphere, the formation of hailstones, the effect of rain in calming the sea, and the action of an oil film on water in preventing waves.

The Kinetics of Gaseous Fluids (1874–1879)

Although not widely recognized or even known about today, this stream of Reynolds' work was of considerable interest to many members of the Victorian scientific community. Featuring in both Volumes 1 and 2 of his collected work (Reynolds 1900, 1901), his papers deal with the force experienced by a surface caused by evaporation, condensation taking place on it or the transfer of heat from it to a gaseous medium, and also the thermally induced transfer of gas through a porous medium. The results of Reynolds' little-known investigation on this last topic are presented in one of the longest and most original of his papers (Reynolds 1879), which extends over 133 pages. In this, he showed by theory and experiment that not only would a difference of pressure cause a gas to flow from one side of a porous plate to the other, but so also would a difference of temperature, even when the pressures on the two sides were equal. To this phenomenon he gave the name "thermal transpiration." Later, he acknowledged the impact of this work on the discoveries made in the first of his turbulent flow papers (Reynolds 1883), observing that "no idea of dimensional properties as indicated by the dependance [sic] of the character of motion on the size of the tube and the velocity of the fluid, occurred to me until after the completion of my investigation on the transpiration of gases."

Advancement in the understanding of the kinetics of fluids in the gaseous state was a notable feature of the science of the 1870s and one to which Reynolds made a singular contribution alongside the established figure of James Clerk Maxwell. Interestingly, The Royal Society's documents on the refereeing of that paper are also of considerable interest and provide illuminating insight into the process by which it finally came to be published; but that is another story.

EARLY DEVELOPMENT OF REYNOLDS' IDEAS ON TURBULENT FLOW

Osborne Reynolds is, without doubt, best known for his papers on fluid flow and turbulence. Besides the two principal papers, several other earlier contributions give insight into the way his ideas were developing. He had been aware of the importance of turbulence in engineering fluid mechanics and heat transfer well before carrying out investigations for his 1883 paper. Some nine years before its appearance, he produced a brief yet far-sighted paper, "On the extent and action of the heating surface of steam boilers" (Paper 14 in Reynolds 1900), in which he pointed out that heat is removed from such a surface not only as a result of molecular action but also by the eddies in the flow, which mixed hotter fluid with cooler fluid. He went on to propose an analogy between heat transfer and skin friction for conditions of (what is today called) turbulent flow. Almost 40 years were to pass before this matter was addressed and extended by others, Prandtl (1910), and Taylor (1916).

In 1877, he described methods for rendering the motions of a fluid visible by means of color bands (Paper 24 in Reynolds 1900). He did so with particular reference to vortex motion, considering vortex lines and vortex rings in several situations. This same visualization technique was used again in his study of laminar and turbulent flow in pipes, which formed part of the investigation considered below (Reynolds 1883).

In Reynolds' presentation to the British Association in 1880, "On the effect of oil in destroying waves on the surface of water" (Paper 38 in Reynolds 1900), he pointed out that, as the wind flowed over the oil film, instead of waves being formed, eddies were produced immediately beneath the oil film (which took on the appearance of "plate glass" and, because of its ability to withstand tension, approximated to a wall boundary condition). Moreover, he ascertained that the eddies were not present in the absence of the film. This observation proved fundamental in guiding his recognition that viscosity played a key role in causing turbulence to be produced in a wall shear flow (as he subsequently made clear in the Introduction of his 1883 paper). Later in the paper he developed his ideas on this matter by presenting the results of an experiment in which two immiscible fluids, one above the other in a horizontal tube, are caused to flow in opposite directions with and without a "skin" or film separating them.

Finally, for completeness, it is noted that, in a short contribution in the year following his 1883 paper entitled "On the two manners of motion of water" (Paper 48 in Reynolds 1901), he compared the characteristics of flow in converging and diverging passages, pointing out that, whereas in the former the conditions were favorable for producing steady flow, in the latter the flow was likely to be eddying and unsteady. This he contrasted with the behavior of flow in tubes that he had recently investigated in such depth and to which attention is now turned. He also discussed the effect of curvature on the flow.

THE TURBULENT FLOW PAPERS AND THEIR PUBLICATION

The 1883 Paper

Although the principal attention here is on the later analytical study (Reynolds 1895), we first consider the experimental investigation (Reynolds 1883), as his discoveries in that paper both shaped the later publication and, moreover, had a significant impact on a referee who was called on to review each of the papers.

Readers will certainly be familiar with the apparatus used in this study, the original print of which has been reproduced in numerous text books. **Figure 1** shows a photograph of the apparatus as it can be seen today at the University of Manchester. The glass tube with flared entry, which is housed within a tank filled with water, still offers a very clear indication of the starkly contrasting states of motion, whether streamline or sinuous (or, in today's terminology, laminar or turbulent). In Reynolds' own words: "The internal motion of water assumes one or other of two broadly distinguishable forms – either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths the most indirect possible." Reynolds' dye-streak studies showed that, for a range of flow velocities, pipe diameters, and viscosities, transition from the former mode to the latter occurred for roughly the same value of the dimensionless parameter that today bears his name.

Figure 1

Osborne Reynolds tank today, The University of Manchester.

The first step in Reynolds' discovery of this parameter appears to have been his observation that "the tendency of water to eddy becomes much greater as the temperature rises." It occurred to him that this might be related to the fact that the viscosity of water diminished as the temperature rose. By examining the governing equations for fluid motion he concluded that the forces involved are of two distinct types, inertial and viscous, and further that the ratio of these terms was related to the product of the mean velocity of the flow and the tube diameter divided by the kinematic viscosity. In his paper he states:

> This is a definite relation of the exact kind for which I was in search. Of course without integration the equations only gave the relation without showing at all in what way the motion might depend upon it. It seemed, however, to be certain, if the eddies were due to one particular cause, that integration would show the birth of eddies to depend on some definite value of [that group of variables].

He recognized, however, that the critical value thus arrived at (sometimes called the "higher critical number") was not unique as it was affected strongly by the level of background disturbances present. In a second series of experiments, he thus set about determining the value of Reynolds number below which highly turbulent motion created at entry to the pipe decayed to laminar flow. In this case, in a different apparatus, he used pressure drop measurements to delineate the mode of flow. Although Reynolds, in that paper, never cited the actual values, Allen (1969) concluded from the figures that he did quote that for the two lead pipes used in this second set of experiments, the "lower critical number" was 2010 and 2060, whereas in his later paper Reynolds (1895) put the critical value between 1900 and 2000.

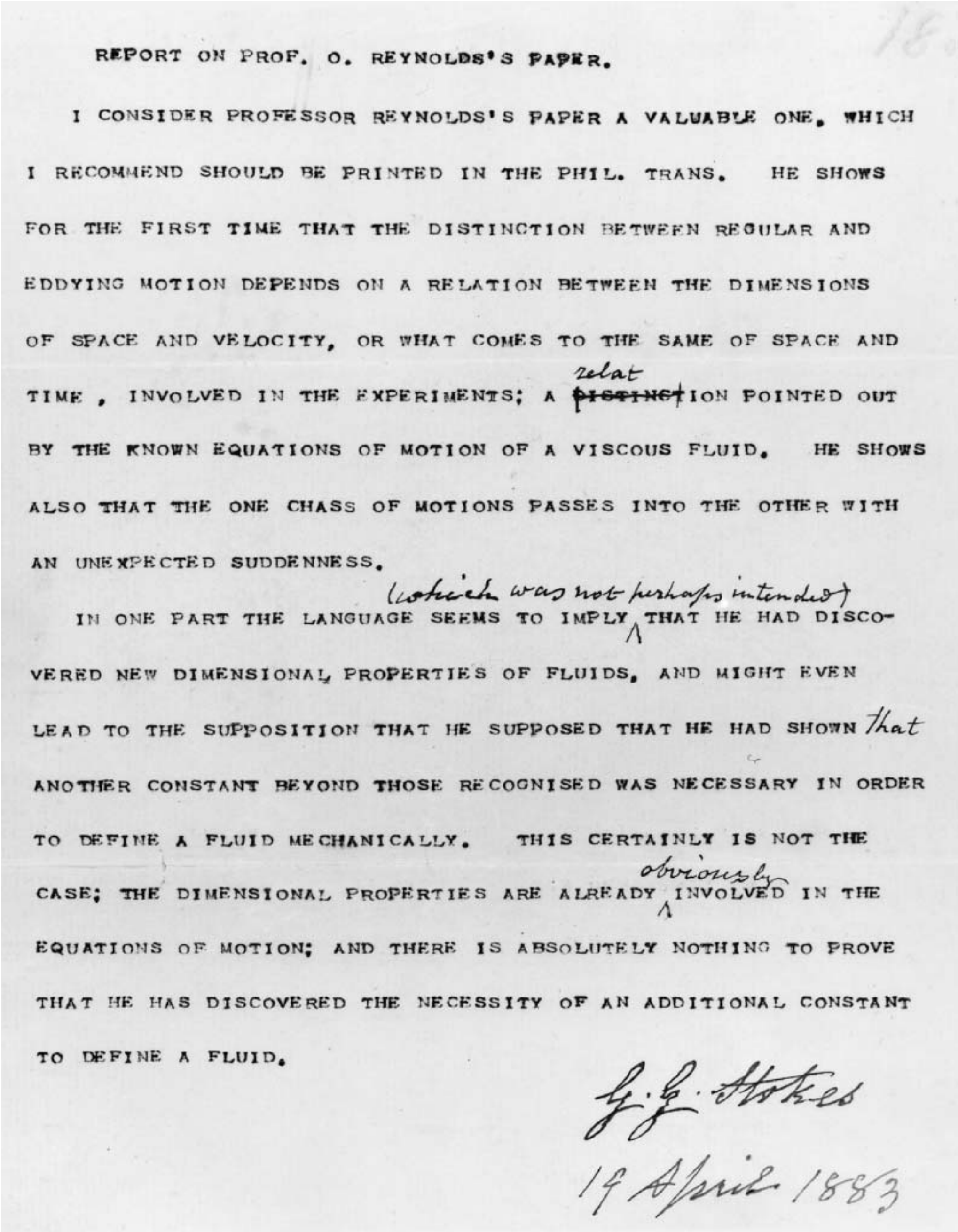
The two referees of the manuscript that Reynolds submitted to The Royal Society were the considerable figures of Sir George Stokes and Lord Rayleigh, each of whom was broadly supportive of publication. Stokes was a pioneer in the use of the typewriter, although it appears that the machine he used for his review had only uppercase letters available (see **Figure 2**), and that the process of typing was sufficiently demanding that Stokes, rather than retyping a final version, chose to insert by hand his subsequent embellishments and corrections (although he failed to correct CHASS in the penultimate line of the first paragraph).

Lord Rayleigh's review (Royal Society Archive Ref. 183) was spread over three pages but amounted to less than 70 words. The first sentence was the lofty, rather patronizing observation:

> This paper records some well contrived experiments on a subject which has long needed investigation – the transition between the laws of flow in capillary tubes and in tubes of large diameter as employed in Engineering. I am of opinion that the results are important, and that the paper should be published in the Phil. Trans.

It then concluded with:

> In several passages the Author refers to theoretical investigations whose nature is not sufficiently indicated. Rayleigh



18.

REPORT ON PROF. O. REYNOLDS'S PAPER.

I CONSIDER PROFESSOR REYNOLDS'S PAPER A VALUABLE ONE, WHICH I RECOMMEND SHOULD BE PRINTED IN THE PHIL. TRANS. HE SHOWS FOR THE FIRST TIME THAT THE DISTINCTION BETWEEN REGULAR AND EDDYING MOTION DEPENDS ON A RELATION BETWEEN THE DIMENSIONS OF SPACE AND VELOCITY, OR WHAT COMES TO THE SAME OF SPACE AND TIME, INVOLVED IN THE EXPERIMENTS; A ~~DISTINCT~~ relatION POINTED OUT BY THE KNOWN EQUATIONS OF MOTION OF A VISCOUS FLUID. HE SHOWS ALSO THAT THE ONE CHASS OF MOTIONS PASSES INTO THE OTHER WITH AN UNEXPECTED SUDDENNESS.

IN ONE PART THE LANGUAGE SEEMS TO IMPLY (which was not perhaps intended) THAT HE HAD DISCOVERED NEW DIMENSIONAL PROPERTIES OF FLUIDS, AND MIGHT EVEN LEAD TO THE SUPPOSITION THAT HE SUPPOSED THAT HE HAD SHOWN that ANOTHER CONSTANT BEYOND THOSE RECOGNISED WAS NECESSARY IN ORDER TO DEFINE A FLUID MECHANICALLY. THIS CERTAINLY IS NOT THE CASE; THE DIMENSIONAL PROPERTIES ARE ALREADY obviously INVOLVED IN THE EQUATIONS OF MOTION; AND THERE IS ABSOLUTELY NOTHING TO PROVE THAT HE HAS DISCOVERED THE NECESSITY OF AN ADDITIONAL CONSTANT TO DEFINE A FLUID.

G. G. Stokes
19 April 1883

Figure 2

Sir George Stokes' review of Reynolds' 1883 paper. Copyright, The Royal Society; Archive Ref. 188.

The paper was duly published and, in the years that followed, each of the referees publicly signaled the exceptional importance of Reynolds' paper. First, Lord Rayleigh, in his 1884 Presidential Address to the British Association in Montreal, paid the following tribute:

> Professor Reynolds has traced with much success the passage from one state of things to the other, and has proved the applicability under these complicated conditions of the general laws of dynamic similarity as adapted to viscous fluids by Professor Stokes. In spite of the difficulties which beset both the theoretical and experimental treatment, we may hope to attain before long to a better understanding of a subject which is certainly second to none in scientific as well as practical interest.

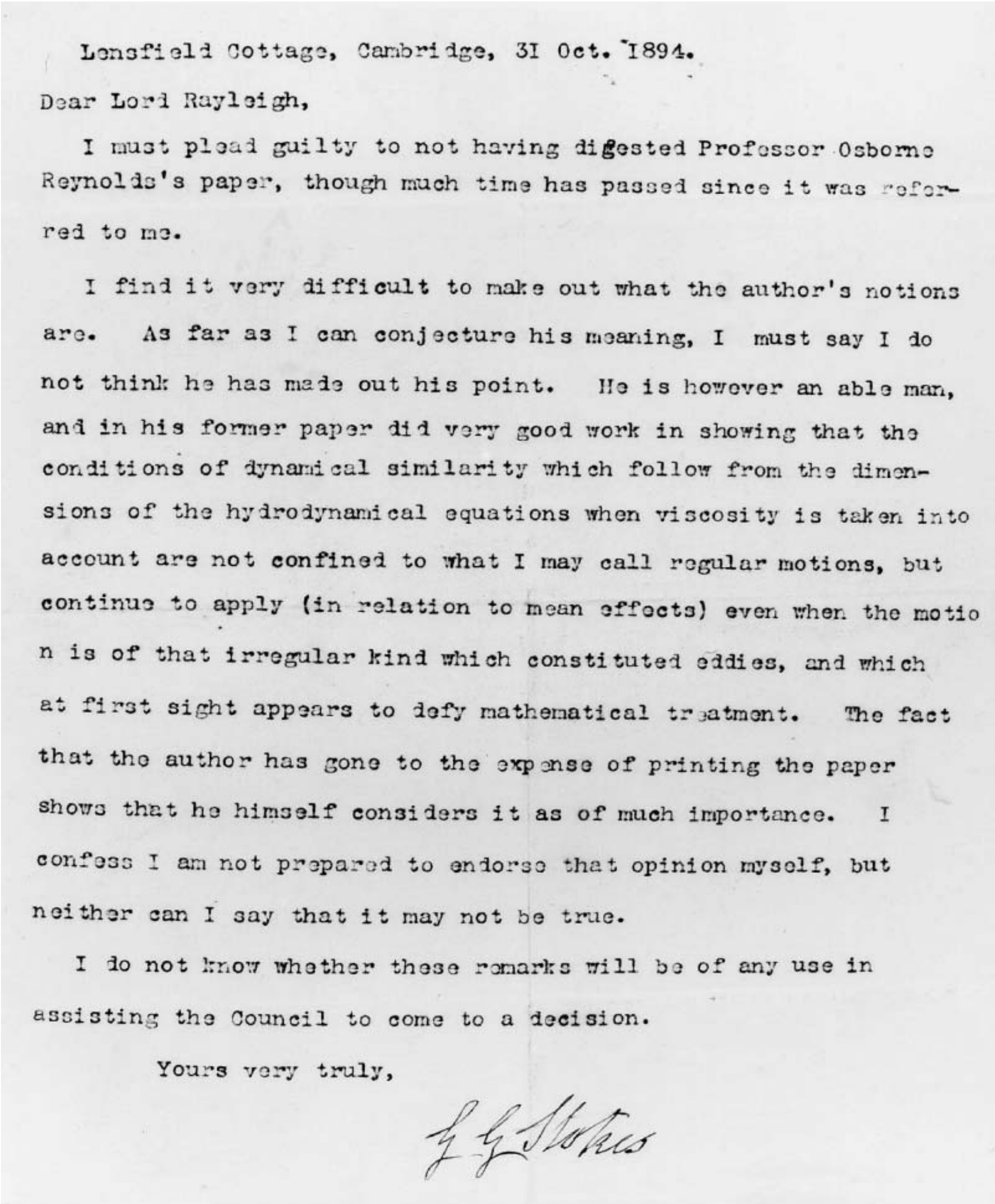
Sir George Stokes served as President of The Royal Society from 1885 to 1890, and in this capacity, in November 1888, he presented the Society's Royal Medal to Osborne Reynolds "for his investigations in mathematical and experimental physics, and on the application of scientific theory to engineering." More than half of Stokes' citation was devoted to a summary of the 1883 paper.

The 1895 Paper

One cannot be sure whether the remarks by Lord Rayleigh, both in his review and in the address to the British Association cited above, provided motivation for Reynolds' work over the years that followed. In any event, a decade later, he felt he was ready to respond and reported orally the results of his extensive analysis to The Royal Society on May 24, 1894. Thereafter he submitted a written version of this work that he had had printed at his own expense to be reviewed for publication in the *Philosophical Transactions of The Royal Society*. By then, Reynolds had held his Chair for more than 25 years, had been a Fellow of The Royal Society for more than 15 and, as noted above, had received the Society's Royal Medal. Thus, he was then arguably the leading engineering fluid mechanicist in England and possibly more widely than that.

By then, Lord Rayleigh had become Editor of the *Philosophical Transactions of The Royal Society*. Perhaps inevitably, on receiving this second manuscript on turbulent flow from Reynolds, he sent it to Sir George Stokes for review. This time, however, the referee's response was very different. After a long period of silence, on October 31, 1894, Sir George, now equipped with a typewriter with both upper- and lowercase letters, sent his reply (see **Figure 3**), effectively acknowledging that he did not understand the work. The letter is a copy-book example of the "on-the-one-hand . . .yet-on-the-other" style of review: Reynolds hadn't made his case—yet, he was an able man and the 1883 paper was sound; moreover, the author had paid to have the present paper printed so obviously *he* thought it was important. However, the reviewer couldn't confirm that view . . . but neither would he assert that it was wrong!

Stokes' concluding sentence (see **Figure 3**) seems to imply that he had finished with the matter, but Lord Rayleigh evidently had other ideas. Although the exchanges are incomplete, it seems that Rayleigh pressed Stokes to go further and when Stokes pleaded that he had mislaid the copy of the paper, he arranged for him to be sent another copy. (Since the paper had been printed, Reynolds had evidently submitted



Lensfield Cottage, Cambridge, 31 Oct. 1894.

Dear Lord Rayleigh,

I must plead guilty to not having digested Professor Osborne Reynolds's paper, though much time has passed since it was referred to me.

I find it very difficult to make out what the author's notions are. As far as I can conjecture his meaning, I must say I do not think he has made out his point. He is however an able man, and in his former paper did very good work in showing that the conditions of dynamical similarity which follow from the dimensions of the hydrodynamical equations when viscosity is taken into account are not confined to what I may call regular motions, but continue to apply (in relation to mean effects) even when the motio n is of that irregular kind which constituted eddies, and which at first sight appears to defy mathematical treatment. The fact that the author has gone to the expense of printing the paper shows that he himself considers it as of much importance. I confess I am not prepared to endorse that opinion myself, but neither can I say that it may not be true.

I do not know whether these remarks will be of any use in assisting the Council to come to a decision.

Yours very truly,

G G Stokes

Figure 3

Sir George Stokes' preliminary assessment of the 1895 paper. Copyright, The Royal Society; Archive Ref. 207.

several copies.) On December 5th, Sir George sent this second copy back indicating that he had now found the copy originally sent to him. He added his regrets that he was "not yet able to go beyond the rough indication contained in a letter sent to Lord Rayleigh some time ago." (Royal Society Archive Ref. 209 from Sir G. G. Stokes to Mr. Rix)

Meanwhile, Lord Rayleigh had sent the paper to a second referee, Horace (later Sir Horace) Lamb, Professor of Mathematics at Manchester, who a decade earlier had been elected a Fellow of the Society. One can only speculate why Rayleigh approached the only other senior fluid mechanicist in Manchester to review his own colleague's work. Nevertheless, on November 21st Lamb sent his longhand assessment, which began with the summarizing statement:

> I think the paper should be published in the Transactions as containing the views of its author on a subject which he has to a great extent created, although much of it is obscure and there are some fundamental points which are not clearly established.

There followed three pages of detailed criticism, including complaints at the inadequate definition of Reynolds' term "mean-mean motion" and a misprint in the manuscript (Royal Society Archive Ref. 208).

There are three further communications from the referees of which only one is dated. Thus, there is some doubt as to the actual sequencing, although the most probable seems to be the following. At some point Sir George Stokes does send his review, a two-page assessment, to Lord Rayleigh, raising some of the problems with the paper he and Lamb had aired earlier. (See **Figure 4** for the first page of the letter. Follow the Supplemental Material link from the Annual Reviews home page at **http://www.annualreviews.org** for the complete **Stokes-Rayleigh letter**.) Thereafter (or, possibly, even before that communication), the referees had made contact with one another, probably through the intervention of Lord Rayleigh, which led Sir George to write (Royal Society Archive Ref. 210) on January 30, 1895:

> Dear Lord Rayleigh,
> I enclose what Lamb meant for a draft of remarks to be submitted to the author. I think we are both disposed to say let the paper be printed, but first let some remarks be submitted to the author. There was very good work in the former paper, and there may be something of importance in this, but the paper is very obscure. In its present state it would hardly be understood.
>
> Yours very truly,
> G.G. Stokes

Follow the Supplemental Material link from the Annual Reviews home page at **http://www.annualreviews.org** to view this "draft of remarks" in Lamb's

Figure 4

The first page of Sir George Stokes' final review of the 1895 paper. Copyright, The Royal Society; Archive Ref. 378. (Follow the Supplemental Material link from the Annual Reviews home page at **http://www.annualreviews.org** for the complete **Stokes-Rayleigh letter**.)

Some remarks about Professor reynolds's paper.

P. 3. (5) a. There is no obscurity about the meaning of u, v, w The word "particle" does **not** mean "molecule", or imply any speculations as to the molecular consttitution of matter. It is simply little part,, and a particle of water, for instance, is simply a portion of water so small that it may be treated as a differential element.

If now we do please to enter into speculations as to the molecular constitution of matter and the motions of the individual molecules, the way in which the equations of motion are ordinarily obtained, combined with the familiar proposition that the centre of gravity of a system will move as if all the mass were collected there and all the forces applied there leads at once to the definition which the author treats as if it were new.

(b) The author says that as a result of this definition the equations are true, and <u>only</u> true, as applied to fluid in which the mean motions, excluding the heat motions, are steady.

Surely the author cannot mean what he says if the word "steady" be taken in its usual sense as applied to the motion of fluids; but if he uses this familiar word in some different sense, it ~~sh~~ should be explained.

What the author would seem to have in view is what has been called stability of motion. But if so, though his idea may be right, the words in which he attempts to express it take as they stand state wg what is not true. In illustration take the dynamical problem of a rough sphere rolling down along an inclined howwlow hollow cylinder, (I) internqlly along the lowest, 9 (2) handwriting. A transcription of this **Lamb-Stokes letter** is provided in Appendix 1, below. From the margin instruction (see online **Lamb-Stokes letter**), which seems to be in Rayleigh's hand, the report was to be copied (meaning that a clerk was to transcribe the review), presumably for onward transmission to Osborne Reynolds.

On receiving the referees' assessment, Reynolds evidently reflected on the criticisms and on February 19th sent his reply to Lord Rayleigh. (Follow the Supplemental Material link from the Annual Reviews home page at **http://www.annualreviews.org** for **Reynolds' response to Rayleigh**, which is also transcribed in Appendix 2, below). It is a remarkable letter partly for its naturalness: its ready admission of the paper's weaknesses accompanied by its ready self-forgiveness: "That I should have scamped the preliminary explanation of this part of the argument and diffused it over the whole paper I can only explain as a consequence of its definite character having blinded me to the difficulties." The opening and penultimate paragraphs also bring out Reynolds' infatuation with long rambling sentences that stand starkly in contrast to Lamb's crisply stated criticisms.

Apparently, no further exchanges between author and editor remain in existence and, as there is no copy of the original manuscript, it is not certain how extensive the changes made actually were. One clear indication of a change in the published version of the paper is that four pages of §5 of the Introduction are placed, without explanation, within square parentheses and end with the date February 18, 1895 (that is, the day preceding Reynolds' sending his response to Lord Rayleigh). Thus, this passage clearly seems to be what Reynolds referred in his reply to Rayleigh as "the full preliminary description of this part of the argument which by permission I shall be glad to substitute for the first two lines of §5. p.3." Since this was the only significant change referred to by Reynolds it appears likely that all other changes were minor, mainly consisting of corrections to typographical errors in the original.

Despite its rather lukewarm reception by the two eminent referees, the paper is seen today as a mighty beacon in the literature of Fluid Mechanics. First and foremost was the decomposition of the flow into mean and fluctuating parts, leading to the averaged momentum equations (now known as the Reynolds equations) in which the Reynolds stresses appear as unknowns. In fact, throughout the analysis, Reynolds treated the averaging in a form akin to what is now known as mass-weighted averaging, 60 years earlier than the source that is usually quoted for introducing that strategy. It was surely just that his experiments had used water as the fluid medium that led to this feature being ignored. The paper's other major analytical result was the turbulent kinetic energy equation on which he observed that the terms comprising products of Reynolds stress and mean velocity gradient represented a transfer of kinetic energy from the mean flow to turbulence. As an indicator of just how far this discovery was ahead of its time, we note that the corresponding, albeit simpler, equation for the mean square temperature fluctuations was not published until the 1950s (Corrsin 1952).

Reynolds' purpose in examining the turbulent kinetic energy equation was to provide an explanation of why the changeover from turbulent to laminar motion should occur at a particular value of the Reynolds number. Indeed, that was the driving rationale for the whole paper. He considered fully developed laminar flow between parallel planes on which a small analytical disturbance was superimposed, which permitted him to obtain expressions for the turbulence energy generation and viscous dissipation rates integrated over the channel. The relative magnitude of these two processes varied with Reynolds number and the lower critical Reynolds number he identified as being that where the overall turbulence energy generation rate had grown to balance the viscous dissipation rate. That his estimates were inaccurate is now seen as irrelevant since the paper contained more than enough novelty for the world of Fluid Mechanics to absorb over the ensuing decades.

APPENDIX 1: TEXT OF JOINT REFEREES' REPORT ON THE 1895 PAPER

Prof Reynolds' Paper

The referees have found great difficulty in following the argument of this paper; partly in consequence of the fact that such terms as "mean-mean motion" and "relative mean motion" are used without any precise definition. There is a well-known distinction between molecular and molar motion; but it is not clear in the case of molar motion how any physical distinction is to be drawn between what is "mean" and what is "relative."

The introduction might be greatly shortened, as a good deal of it can only be understood after reading the rest of the paper. The purport of §5(a) p.3 is not evident. The author's view does not appear to be different from that generally held, but it is insisted upon as something new.

The statement, in §5(b), that the ordinary equations of a viscous fluid are true only when the motion is approximately steady, is questionable. It is perhaps based on the investigation on p.9; but this is purely mathematical; and there is besides a difficulty in seeing the connection between equations (7) and (8A). It would seem as if there had been a slip in writing $\underline{u}$ for $\underline{\bar{u}}$; but at any rate there is need of explanation. It is to be noted that the argument, if valid, would show that there are <u>geometrical</u> difficulties in the way of applying the idea of mean velocity to cases other than steady homogeneous motion.

The essence of the paper lies in the equations on pp. 15, 16†. If these are clearly established a great point would be secured, but its reasoning is somewhat obscure, and needs much amplification. The conception of 'mean-mean-motion' is a very delicate one and it is not made evident in what sense $\bar{u}$, $\bar{v}$, $\bar{w}$ are continuous functions, or on what conditions the derivatives $d\bar{u}/dx$, etc. are supposed to be formed. The whole argument turns on questions of this kind, and it is just here that explanations are wanting.'

†Authors' footnote: Taking account of the 4-page insert made by Reynolds in the published version, the reference here is to Equations (13–19).

APPENDIX 2: OSBORNE REYNOLDS' RESPONSE

Dear Lord Rayleigh,

From the copy of the remarks on my paper on the criterion, which you have sent me, it is clear that the referees have found great difficulty in understanding the drift of the main argument; namely that which relates to the geometrical separation of the components $\underline{u}$, $\underline{v}$, $\underline{w}$, at each point of a system into mean-components $\underline{\bar{u}}$, $\underline{\bar{v}}$, $\underline{\bar{w}}$ and relative components $\underline{u}'$, $\underline{v}'$, $\underline{w}'$ and as to the conditions of distribution of $\bar{u}$, $\bar{v}$, $\bar{w}$ under which such separation is possible.

I am very glad to know of these difficulties and of the opportunity it afforded me of improving the paper in this particular.

As it is by such separation of the simultaneous component of velocity at each point, introduced into the equations of viscous fluid, that the evidence of a geometrical limit to the criterion appears independently of all physical considerations, any want of clearness on this point, no doubt, confuses the whole argument.

That I should have scamped the preliminary explanation of this part of the argument and diffused it over the whole paper I can only explain as a consequence of its definite character having blinded me to the difficulties which would thereby result in distinguishing what was new from what was already accepted, and of my desire to set forth the proof of the actual maintenance of the geometrical conditions under which such separation is possible afforded by experiment, as well as to indicate the general character of the mechanical-actions, expressed in the equations of motion, on which such maintenance depends.

I now enclose you in M.S.S. a full preliminary description of this part of the argument which by permission I shall be glad to substitute for the first two lines of §5 p.3. It contains, what I hope will be found, a clear definition of the terms mean-mean motion and relative-mean motion as well as of mean-motion and heat-motions and of the geometrical distinctions between these motions. And although no physical-distinction between mean-molar and relative-molar is draw[n] other than what is implied by the geometrical distinction that the integrals of $\rho\underline{\bar{u}}$, etc, taken over the space determined by the scale or period-in-space of the relative mean motion $\rho\underline{u}'$, etc, are the components of momentum of the molar motion of the mechanical system within S while the integrals of $\rho\underline{u}$ [should apparently be $\rho\underline{u}'$], etc, taken over the same space are zero, it is shown that such physical distinction has no place in the argument any further than it is sup[p]ressed by the terms in the equations of motion.

With reference to the difficulties in logic of §8 p.9, equations 7 and 8a, this is intirely removed by replacing the bar ($\underline{\bar{u}}$) which has dropped from the $\underline{u}$ in the left of Equation 4, p8.

There are, I am sorry to say, certain other misprints in the paper which must have increased the inherent difficulties of the subject.

Very truly yours,
Osborne Reynolds

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Contents

Indexes

Errata

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